Centrum Wiskunde & Informatica

A projection-based partitioning for tomographic reconstruction

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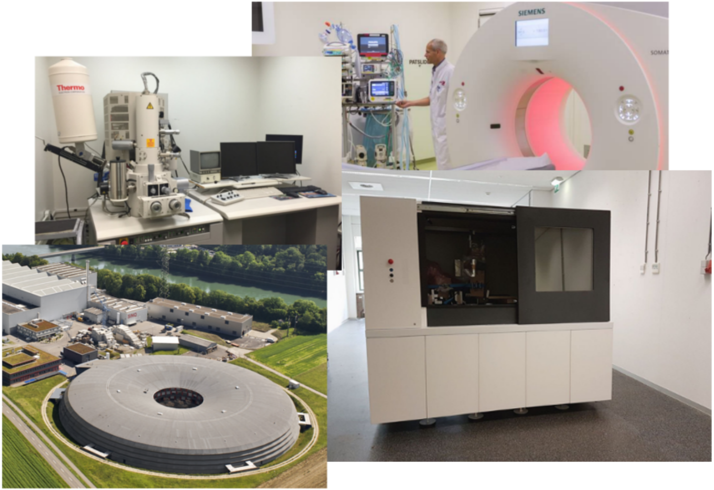
Outline

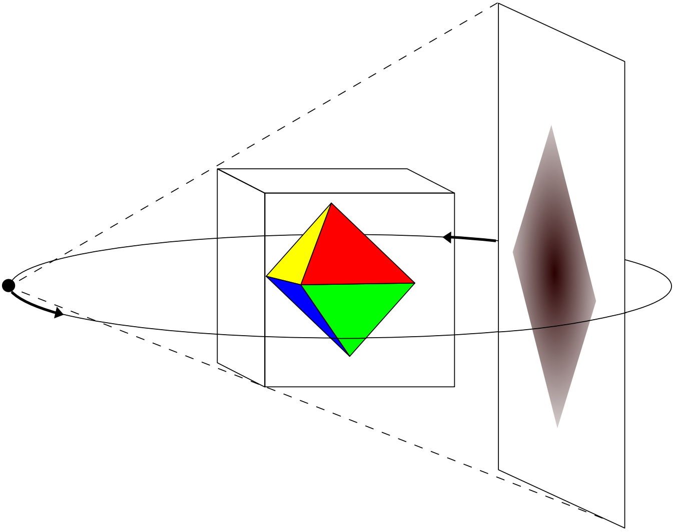
- Tomography, partitioning problems in imaging
- Previous work: GRCB algorithm
- Communication volume, shadows and overlaps
- Continuous model for load balancing
- Communication data structures
- Results and conclusion

Background

Tomography applications



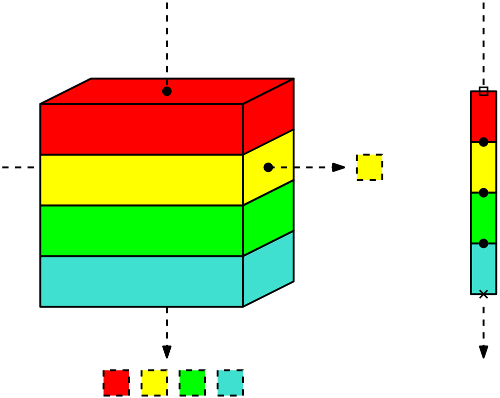
Tomography



Reconstruction problem

- TODO big data sets, typical sizes, different acquisition geometries
- Distributed 3D volume over many GPUs, minimizing communication

Communication in tomography



- Each combination source position and detector pixel defines a ray, in the solver each ray is traced through the discretized 3D volume
- Tomographic reconstruction problem deal with anywhere between 10^9 and 10^{11} rays

Partitionings in tomography

- Partition 3D volume while minimizing the *line cut*
- The line cut is the number of additional parts a line crosses
- Assigning the entire volume to a single GPU is still a partitioning. Good for communication, but defeats the purpose.
- The load of a *voxel* is the number of rays crossing it. The load of a part is the sum over the loads of its voxels.
- A good partitioning ensures that each part has a similar load.

Previous work

Problem (Tomographic partitioning)

Let V be a cuboid, and G a set of rays through V. Find a p-way partitioning of V, that minimizes the total line cut, while ensuring that the parts have a roughly equal load.

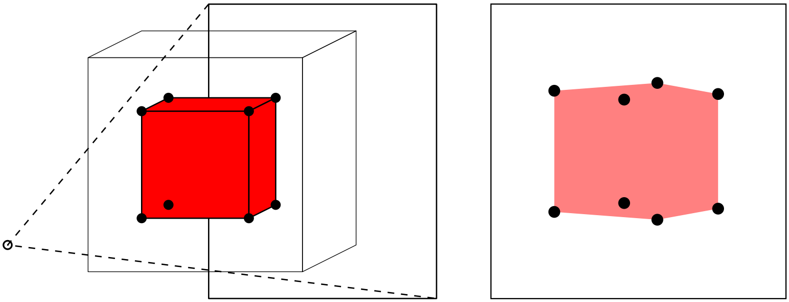
- *Recursive bisectioning strategy*: recursively split V in two, somewhere along one of the three axes.
- It is possible to find the best partitioning of this kind in $\mathcal{O}(p|G|\log|G|)$ time (GRCB algorithm).
- Communication reduced by between 60% and 90%
- Each GPU guaranteed to perform the same amount of work

A geometric partitioning method for distributed tomographic reconstruction.
JWB, Rob Bisseling, Joost Batenburg. Parallel Computing, 2019.
doi:10.1016/j.parco.2018.12.007

Projection-based partitioning

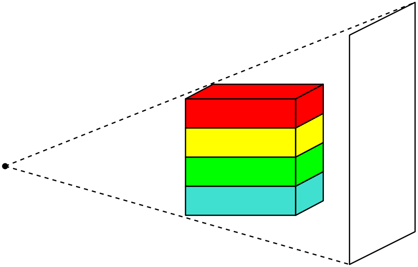
Shadows

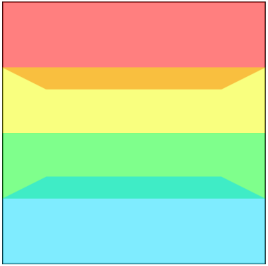
- *Reducing the input size*: look at projections instead of rays.



Shadow overlap

- Communication volume is proportional to area of the shadow overlaps of parts.





Algorithm sketch

Subroutine: $\textsc{communicationVolume}$

Input: V_L, V_R, projection set Π

Output: communication volume Θ

$\Theta \leftarrow 0$

for all $\pi \in \Pi$ **do**

$$\text{shadow}_L \leftarrow \underbrace{\textsc{convexHull}}_{2}(\underbrace{\textsc{project}}_{1}(\pi, \textsc{corners}(V_L)))$$

$$\text{shadow}_R \leftarrow \textsc{convexHull}(\textsc{project}(\pi, \textsc{corners}(V_R)))$$

$$\Theta \leftarrow \Theta + \underbrace{\textsc{area}}_{4}(\underbrace{\text{shadow}_L \cap \text{shadow}_R}_{3})$$

if consider gradient **then**

$$\Theta \leftarrow \Theta + M \times \textsc{area}(V_L \cap V_R)$$

Continuous load balance

- If we have a candidate partitioning, we can efficiently estimate the communication volume using the part shadows.
- Generating candidate partitionings involve finding a projection-based estimate for the *load*. (Number of rays crossing voxels).
- Estimate by integrating over ray densities for each source point. Find c such that:

$$\int_{x_1}^{c} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \sum_{k=1}^{|\Pi|} \frac{1}{||\vec{x} - \vec{s}_k||_2^2} \, dz \, dy \, dx$$
$$= \int_{c}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} \sum_{k=1}^{|\Pi|} \frac{1}{||\vec{x} - \vec{s}_k||_2^2} \, dz \, dy \, dx.$$

Equal load

- We can reduce the integral to 2D, and then solve numerically:

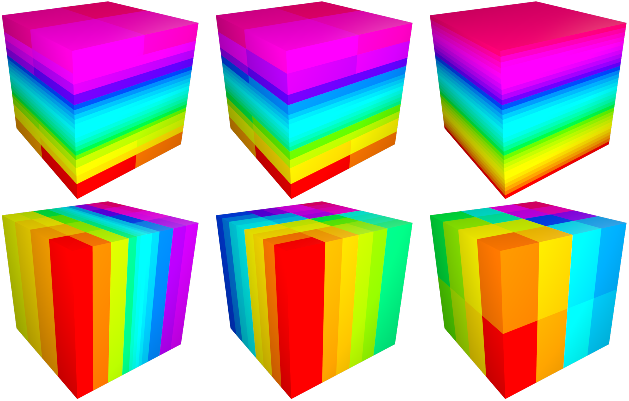
$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\sum_{k=1}^{|\Pi|}\left(\frac{1}{a_k(x,y)}\left(\arctan\left(\frac{z_2-s_{k,z}}{a_k(x,y)}\right)\right.\right.$$
$$\left.\left.-\arctan\left(\frac{z_1-s_{k,z}}{a_k(x,y)}\right)\right)\right)\,dy\;dx, \tag{1}$$

  where

$$a_k(x,y)=\sqrt{(x-s_{k,x})^2+(y-s_{k,y})^2}.$$

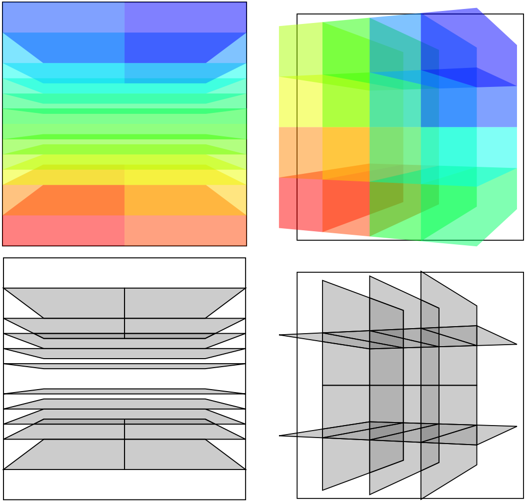
- For certain acquisition geometries, need to consider the cone instead instead of the entire cuboid. We reject samples outside cone.
- Most successful strategy we found so far is an adaption of a standard streaming median find algorithm.

Partitioning results



- *Partitioning method*: Use continuous load balance to find candidate splits in each direction, use shadow characterization of the communication volume to choose the best split. Recurse on the subvolumes.

Communication data structures



- Overlap structures: finding (possibly non-simple, non-convex) polygons for each set of contributors.

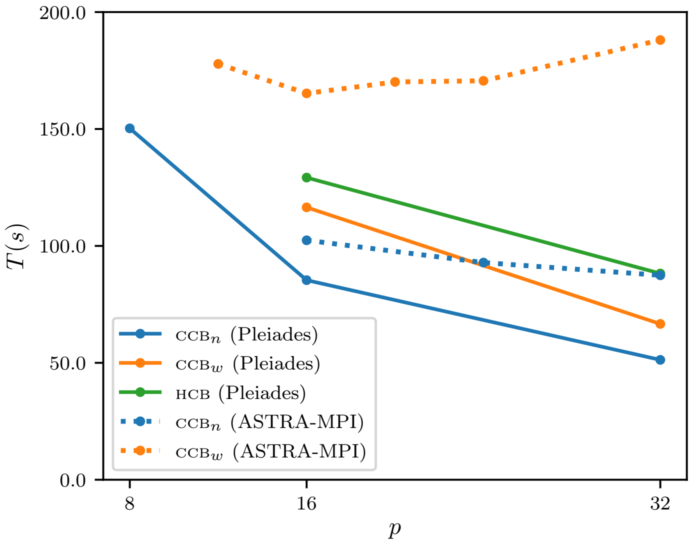
Overlap algorithm

Subroutine: $\textsc{FindFaces}$
Input: $\pi = \{V_s\}, \pi_k$
Output: $\textsc{overlay}$

$\textsc{overlay} \leftarrow \textsc{EmptyArrangement}$
for $0 \leq s < p$ **do**
  $\textsc{shadow}_s \leftarrow \textsc{convexHull}(\textsc{project}(\pi_k, \textsc{vertices}(V_s)))$
  $\textsc{arrangement}_s \leftarrow \textsc{FromFaceTag}(\textsc{shadow}_s, [s])$
  $\textsc{merge}(\textsc{overlay}, \textsc{arrangement}_s, \textsc{concatenate})$

- *Subdivision merging* algorithms: "find area on map with forests, low precipitation, high temperature".
- We rasterize the resulting faces, and perform aggregrate reads from GPU textures containing image data for communication between nodes

Reconstruction times



Conclusion

- TODO Faster, equal results