Bulk-synchronous pseudo-streaming for many-core accelerators

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Overview

Parallella

Epiphany BSP

Extending BSP with streams

Examples

- Inner product
- Matrix multiplication
- Sort

Parallella

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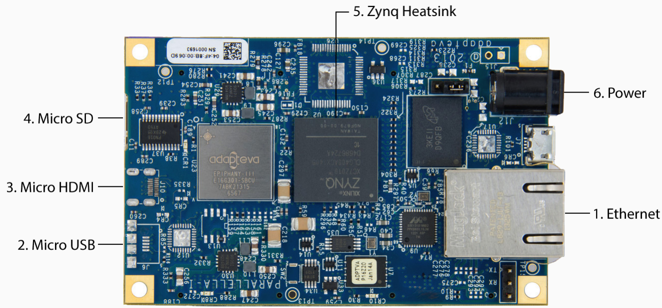

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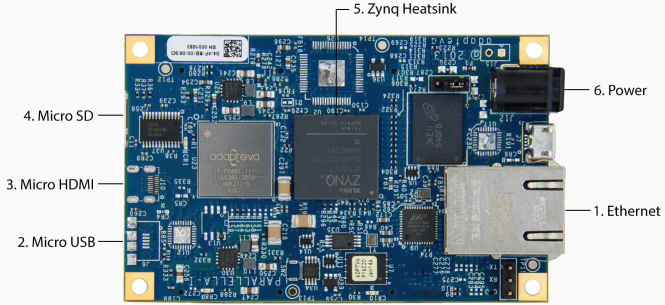

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Epiphany co-processor

- $N \times N$ grid of RISC processors, clocked by default at 600 MHz (current generations have 16 or 64 cores), each with *limited local memory*.
- Efficient communication network with '*zero-cost start up*' communication. Asynchronous connection to *external memory pool* using DMA engines (used for software caching).
- Energy efficient @ 50 GFLOPs/W (single precision), in 2011, top GPUs about $5\times$ less efficient.

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Epiphany memory

- Each Epiphany core has 32 kB of **local memory**, on 16-core model 512 kB available in total. There are no caches.
- On each core, the kernel binary and stack already take up a large section of this memory.
- On the Parallella, there is 32 MB of **external RAM** shared between the cores, and 1 GB of additional RAM accessible from the ARM host processor.

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Hello World: ESDK (124 LOC)

```
// host

const unsigned ShmSize = 128;
const char ShmName[] = "hello_shm";
const unsigned SeqLen = 20;

int main(int argc, char *argv[])
{
    unsigned row, col, coreid, i;
    e_platform_t platform;
    e_epiphany_t dev;
    e_mem_t    mbuf;
    int rc;

    srand(1);

    e_set_loader_verbosity(H_D0);
    e_set_host_verbosity(H_D0);

    e_init(NULL);
    e_reset_system();
    e_get_platform_info(&platform);

    rc = e_shm_alloc(&mbuf, ShmName,
          ShmSize);
    if (rc != E_OK)
        rc = e_shm_attach(&mbuf, ShmName
              );
    // ...
```

```
// kernel

int main(void) {
    const char          ShmName[] = "
         hello_shm";
    const char          Msg[] = "Hello 
         World from core 0x%03x!";
    char                buf[256] = { 0 };
    e_coreid_t               coreid;
    e_memseg_t               emem;
    unsigned            my_row;
    unsigned            my_col;


    // Who am I? Query the CoreID from
         hardware.
    coreid = e_get_coreid();
    e_coords_from_coreid(coreid, &my_row
         , &my_col);

    if ( E_OK != e_shm_attach(&emem,
         ShmName) ) {
        return EXIT_FAILURE;
    }

    snprintf(buf, sizeof(buf), Msg,
         coreid);
    // ...
```

Hello World: Epiphany BSP (18 LOC)

```c
// host

#include <host_bsp.h>
#include <stdio.h>

int main(int argc, char** argv) {
    bsp_init("e_hello.elf", argc, argv);

    bsp_begin(bsp_nprocs());

    ebsp_spmd();

    bsp_end();

    return 0;
}
```

```c
// kernel

#include <e_bsp.h>

int main() {
    bsp_begin();

    int n = bsp_nprocs();
    int p = bsp_pid();

    ebsp_printf("Hello world from core %
        d/%d", p, n);

    bsp_end();

    return 0;
}
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BSP computers

- The BSP model [Valiant, 1990] describes a general way to perform parallel computations.
- An abstract BSP computer is associated to the model that has p processors, which all have access to a communication network.

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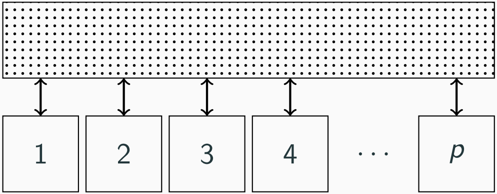

BSP computers (cont.)

- BSP programs consist of a number of supersteps, that each have a computation phase, and a communication phase. Each superstep is followed by a barrier synchronisation.
- Each processor on a BSP computer has a processing rate r. It has two parameters: g, related to the communication speed, and l the latency.
- The running time of a BSP program can be expressed in terms of these parameters! We denote this by $T(g, l)$.

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BSP on low-memory

- Limited local memory, *classic* BSP programs can not run.
- Primary goal should be to minimize communication with external memory.
- Many known performance models can be applied to this system (EM-BSP, MBSP, Multi-BSP), **no portable way to write/develop algorithms**.

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BSP accelerator

- We view the Epiphany processor as a BSP computer with **limited local memory** of capacity L.
- We have a **shared external memory** unit of capacity E, from which we can read data asynchronously with **inverse bandwidth** e.
- Parameter pack: (p, r, g, l, e, L, E).

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Parallella as a BSP accelerator

- $p = 16$, $p = 64$
- $r = (600 \times 10^6)/5 = 120 \times 10^6 \text{ FLOPS}^{(*)}$
- $l = 1.00 \text{ FLOP}$
- $g = 5.59 \text{ FLOP/word}$
- $e = 43.4 \text{ FLOP/word}$
- $L = 32 \text{ kB}$
- $E = 32 \text{ MB}$

(*): In practice one FLOP every 5 clockcycles, in theory up to 2 FLOPs per clockcycle.

Extending BSP with streams

External data access: streams

- *Idea:* present the input of the algorithm as **streams** for each core. Each stream consists of a number of **tokens**.
- The ith stream for the sth processor:

$$\Sigma_i^s = (\sigma_1, \sigma_2, \ldots, \sigma_n)$$

- Tokens fit in local memory: $|\sigma_i| < L$.
- We call the BSP programs that run on the tokens loaded on the cores hypersteps.

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Structure of a program

- In a hyperstep, while the computation is underway, the next tokens are loaded in (asynchronously).
- The time a hyperstep takes is either **bound by bandwidth or computation**.
- Cost function:

$$\tilde{T} = \sum_{h=0}^{H-1} \max\left(T_h, e \sum_i C_i\right).$$

Here, C_i is the token size of the ith stream, and T_h is the (BSP) cost of the hth hyperstep.

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Pseudo-streaming

- In video-streaming by default the video just 'runs'. But viewer can skip ahead, rewatch portions. In this context referred to as pseudo-streaming.
- Here, by default the next logical token is loaded in. But programmer can *seek* within the stream.
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BSPlib extension for streaming

```
// host
void* bsp_stream_create(
        int processor_id,
        int stream_size,
        int token_size,
        const void* initial_data);

// kernel
int bsp_stream_open(int stream_id);
void bsp_stream_close(int stream_id);
```

BSPlib extension for streaming (2)

```
int bsp_stream_move_down(
        int stream_id,
        void** buffer,
        int preload);

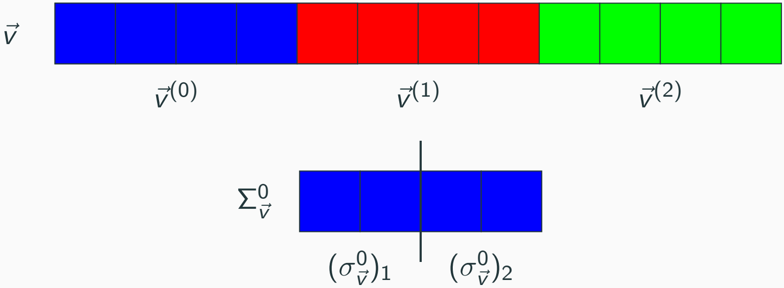
int bsp_stream_move_up(
        int stream_id,
        const void* data,
        int data_size,
        int wait_for_completion);

void bsp_stream_seek(
        int stream_id,
        int delta_tokens);
```

Examples

Example 1: Inner product

- *Input*: vectors $\vec{v}, \vec{u}$ of size n
- *Output*: $\vec{v} \cdot \vec{u} = \sum_i v_i u_i$.



Example 1: Inner product (cont.)

- *Input*: vectors $\vec{v}, \vec{u}$ of size n
- *Output*: $\vec{v} \cdot \vec{u} = \sum_i v_i u_i$.

1. Make a p-way distribution of $\vec{v}, \vec{u}$ (e.g. in blocks), resulting in subvectors $\vec{v}^{(s)}$ and $\vec{u}^{(s)}$.
2. These subvectors are then split into tokens that each fit in L. We have two streams for each core s:

$$\Sigma_{\vec{v}}^{s} = ((\sigma_{\vec{v}}^{s})_1, (\sigma_{\vec{v}}^{s})_2, \ldots, (\sigma_{\vec{v}}^{s})_H),$$

$$\Sigma_{\vec{u}}^{s} = ((\sigma_{\vec{u}}^{s})_1, (\sigma_{\vec{u}}^{s})_2, \ldots, (\sigma_{\vec{u}}^{s})_H).$$

3. Maintain a partial answer α_s throughout the algorithm, add $(\sigma_{\vec{v}}^{s})_h \cdot (\sigma_{\vec{u}}^{s})_h$ in the hth hyperstep. After the final tokens, sum over all α_s.

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Example 2: Matrix multiplication

- *Input*: Matrices A, B of size $n \times n$
- *Output*: $C = AB$

We decompose the (large) matrix multiplication into smaller problems that can be performed on the accelerator (with $N \times N$ cores). This is done by decomposing the input matrices into $M \times M$ **outer blocks**, where M is chosen suitably large.

$$AB = \left(\begin{array}{c|c|c|c} A_{11} & A_{12} & \dots & A_{1M} \\ \hline A_{21} & A_{22} & \dots & A_{2M} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline A_{M1} & A_{M2} & \dots & A_{MM} \end{array}\right) \left(\begin{array}{c|c|c|c} B_{11} & B_{12} & \dots & B_{1M} \\ \hline B_{21} & B_{22} & \dots & B_{2M} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline B_{M1} & B_{M2} & \dots & B_{MM} \end{array}\right)$$

Example 2: Matrix multiplication (cont.)

We compute the **outer blocks** of C in row-major order. Since:

$$C_{ij} = \sum_{k=1}^{M} A_{ik} B_{kj},$$

a complete outer block is computed every M hypersteps, where in a hyperstep we perform the multiplication of one outer blocks of A, and one of B.

Each block is again decomposed into **inner blocks** that fit into a core:

$$A_{ij} = \left(\begin{array}{c|c|c|c} (A_{ij})_{11} & (A_{ij})_{12} & \ldots & (A_{ij})_{1N} \\ \hline (A_{ij})_{21} & (A_{ij})_{22} & \ldots & (A_{ij})_{2N} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline (A_{ij})_{N1} & (A_{ij})_{N2} & \ldots & (A_{ij})_{NN} \end{array}\right).$$

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Example 2: Matrix multiplication (cont.)

The streams for core (s, t) are the inner blocks of A that belong to the core, laid out in row-major order, and the inner blocks of B in column-major order.

$$
\begin{aligned}
\Sigma_{st}^{A} = & \underbrace{(A_{11})_{st}(A_{12})_{st}\dots(A_{1M})_{st}}_{\circlearrowright\ M \text{ times}}\ \underbrace{(A_{21})_{st}(A_{22})_{st}\dots(A_{2M})_{st}}_{\circlearrowright\ M \text{ times}} \\
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In a hyperstep a suitable BSP algorithm (e.g. Cannon's algorithm) is used for the matrix multiplication on the accelerator.

We show that the cost function can be written as:

$$\tilde{T}_{\text{cannon}} = \max\left(2\frac{n^3}{N^2} + \frac{2Mn^2}{N}g + NM^3 l,\ 2\frac{Mn^2}{N^2}e\right).$$

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Example 3: Sorting

- *Input*: An array A of comparable objects.
- *Output*: The sorted array $\tilde{A}$.

1. Parallel bucket sort: create p buckets, put each element of A in the appropriate bucket, let the sth core sort the sth bucket.
2. **Sample sort** samples elements of A in order to balance the buckets.

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Sorting: Splitters

1. Split the input array to create p equally sized streams. Also create p initially empty streams that will be the buckets.
2. We adapt the sample sort algorithm, first we need to find the buckets, which is **Phase 1** of our algorithm.
3. Each core samples k elements randomly from its stream. We do this using a classic streaming algorithm called *reservoir sampling*. These samples are then sorted.

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Sorting: Splitters (cont.)

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- In **Phase 2** of the algorithm we fill the buckets with data.
- In a hyperstep, we run a BSP sort on the current tokens. Next, each core will have consecutive elements that can be sent to the correct buckets efficiently.
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Summary

- Parallella and the Epiphany: great platform for BSP.
- Pseudo-streaming algorithms are a convenient way to think about algorithms for this platform.
- Can often (re)use BSP algorithms, and generalize them to this streaming framework, even if local memory is limited.

Thank you for your attention. Questions?

Sources

1. Parallella, Adapteva Epiphany: `http://www.adapteva.org`
2. Epiphany BSP: `http://www.codu.in/ebsp`
3. KiloCore: `https://www.ucdavis.edu/news/worlds-first-1000-processor-chip`